

Finite Field-dependent Symmetry in Thirring Model

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In this paper, we consider a D -dimensional massive Thirring model with ($2 < D < 4$). We derive an extended BRST symmetry of the theory with finite field-dependent parameter. Further we compute the Jacobian of functional measure under such an extended transformation. Remarkably, we find that such Jacobian extends the BRST exact part of the action which leads to a mapping between different gauges. We illustrate this with the help of Lorentz and R_ξ gauges. We also discuss the results in Batalin-Vilkovisky framework.

I. INTRODUCTION

From earlier studies in quantum field theories (QFT), it is found that ground states showing sensitivity to the number of light fermion flavors are very important. One of the major example of such theories is the Thirring model, a quantum field theory of fermions interacting via a conserved vector current term, described in three-dimensional space-time. Thirring model has been studied as a candidate for the scenario of the fermion dynamical mass generation [1–3]. It is important because the fermion dynamical mass generation is the central issue of the dynamical electroweak symmetry breaking such as the technicolor [4] and the top quark condensate [5]. The role of four fermion interaction in the context of walking technicolor [6] and strong ETC technicolor [7] has also been investigated. In particular, the scalar/pseudoscalar-type four fermion interactions with the gauge interaction have played a very important role in $D = 4$ dimensions as a renormalizable model [8]. This model with gauge interaction is known as gauged Nambu-Jona-Lasinio (NJL) model [9–11]. It has been observed that the phase structure of such a gauged NJL model in $D = 4$ dimensions is quite similar to that of the $D = 3$ dimensional scalar/pseudoscalar-type four fermion theory without gauge interactions [12], called as Gross-Neveu model [13]. The gauged Thirring model, a natural gauge invariant generalization of the Thirring model, has been studied in [14], where it is shown that, in the strong gauge-coupling limit, the gauged Thirring model reduces to the proposed reformulation of the Thirring model [2] as a gauge theory.

To quantize the gauge theories, the BRST formulation [15–18] is a powerful method, which guarantees the renormalizability and unitarity of gauge theories. In [3] certain aspects of BRST quantization for Thirring model in ($2 < D < 4$) dimensions are discussed. This model is described in Lorentz gauge and R_ξ gauges there. For instance, it is well-known that, in R_ξ gauge, the Stueckelberg (also called Batalin-Fradkin) field θ is completely decoupled to the massive vector boson independently of ξ . This would lead to simplicity in performing a numerical computations. However, for Lorentz gauge, the Stueckelberg field θ is decoupled to the massive vector boson only for $\xi = 0$ (which refers Landau gauge). These two gauges have their own advantages. A mapping between these two gauges in the perspective of the Thirring model would be remarkable because if one gets a complicated expression for the calculations in one gauge, then this mapping would be very helpful. In this context, we try to achieve this goal with the help of generalization of BRST quantization and with the Batalin-Vilkovisky (BV) description.

The key idea of generalization (extension) of BRST symmetry is to make the parameter of transformation finite and field-dependent in certain way, known as finite field-dependent BRST (FFBRST)

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transformation [19]. The generalization, in this way, has found enormous applications in wide area of gauge theories [20, 21] as well as in gravity theory [22]. For example, the celebrated Gribov issue [23–25] in Yang-Mills theory has been addressed in the framework of FFBRST formulation (for details see refs. therein [26]). The FFBRST transformations have been emphasized in higher-form gauge theories, an important ingredient of string theories [27]. Further, for the superconformal Chern-Simmons- matter theories [28–30], the aspects of the generalized BRST symmetry have also reported in [31–33]. The validity of such generalization has been established at quantum level also [34, 35] with the help of the BV formulation [36]. Recently, the FFBRST formulation has been studied in (topological) lattice sigma models [37]. A slightly different field-dependent BRST formulation has also been made, in Yang-Mills theories also [38], which involves a linear dependence on the corresponding Grassmann-odd parameter, naturally, without having recourse to any quadratic dependence. Since Ref. [38] does not deal with the case of BRST-antiBRST symmetry, and so any non-trivial quadratic dependence on the transformation parameters cannot occur. Moshin and Reshetnyak, in Ref. [39], incorporates, systematically, the case of BRST-antiBRST symmetry in Yang-Mills theories within the context of finite transformations, which deals with the case of a quadratic dependence on the corresponding parameters for two reasons: 1) finite BRST-antiBRST symmetry does admit a non-trivial quadratic dependence on two different Grassmann-odd parameters, 2) this dependence actually turns out to be necessary for a systematic treatment of finite BRST-antiBRST transformations. Further, the concept of finite BRST-antiBRST symmetry to the case of general gauge theories has been extended in Refs. [40, 41], whereas Ref. [42] by the same authors generalizes the corresponding parameters to the case of arbitrary Grassmann-odd field-dependent parameters, as compared to the so-called “potential” form of parameters used in the previous articles [39–41].

The field theoretic models, with fermion interactions of current-current type, are not renormalizable in $D = 4$ as the coupling constant takes the dimension of mass inverse square. Nevertheless, it has been established that a class of D ($2 < D < 4$) dimensional four-fermion model is renormalizable in the different expansion scheme [43]. In this paper, we consider a renormalizable D ($2 < D < 4$) dimensional gauge non-symmetric Thirring model to discuss the various gauge connection. After the introduction of the auxiliary field, the theory still remains gauge non-invariant. Of course, the Thirring model can be rewritten into the massive vector theory with which the fermion couples minimally. First of all, we discuss the gauge invariant version of the model, which can be quantized correctly only after breaking the local gauge invariance. This is achieved by fixing the gauges specifically. For the present case, the Lorentz and R_ξ gauges are considered. We write corresponding gauge fixed Faddeev-Popov action. The resulting action, by summing the classical action to the gauge-fixed action, remains invariant under the BRST symmetry. Further, we generalize the BRST symmetry, by forming the transformation parameter finite and field dependent. The generalized BRST symmetry leaves the Faddeev-Popov action invariant. The only difference, with the usual BRST symmetry, is the functional measure, which is not covariant under the generalized BRST transformation. So, we compute the Jacobian for functional measure and found that it depends, explicitly, on the infinitesimal field-dependent parameter. Then, the different value of the field-dependent parameter will lead to different contribution in the generating functional. Here, we show that, for a particular value of such parameter, Jacobian switches the generating functional from one gauge to another gauge. We illustrate this result for a particular set of gauges, namely, the Lorentz gauge and R_ξ gauge. Further, we establish the result at quantum level, by mapping the solutions of quantum master equation in BV framework.

The paper is organized as follows. In section II, we discuss the BRST quantization of Thirring model, with an arbitrary as well as specific gauge choices in D ($2 < D < 4$) dimensions. Then, we derive methodology for extended BRST symmetry for Thirring model, where Jacobian for path integral measure is computed explicitly. The connections of various gauges through this extended BRST formulation are described in section IV. In section V, we establish the mapping of different gauges in BV formulation. The last section summarizes the present investigations with future motivations.

II. THIRRING MODEL: BRST SYMMETRY

In this section, we analyse the Thirring model D ($2 < D < 4$) dimensions. The Thirring model is given by the Lagrangian density

$$\mathcal{L}_{Th} = \bar{\psi}^a i \gamma^\mu \partial_\mu \psi^a - m \bar{\psi}^a \psi^a - \frac{G}{2N} (\bar{\psi}^a \gamma^\mu \psi^a)^2. \quad (1)$$

Here, ψ^a refers a Dirac fermion with flavor index a which runs from 1 to N . The gamma matrices γ_μ ($\mu = 0, 1, 2, \dots, D-1$) satisfy the Clifford algebra, following $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \mathbf{1}$.

The Lagrangian can, further, be rewritten in terms of an auxiliary vector field A_μ as

$$\mathcal{L}_{Th'} = \bar{\psi}^a i \gamma^\mu (\partial_\mu - \frac{i}{\sqrt{N}} A_\mu) \psi^a - m \bar{\psi}^a \psi^a + \frac{1}{2G} A_\mu^2, \quad (2)$$

which coincides with (1) when we perform equation of motion of A_μ . Here, we note that the field A_μ is just a vector field which represents the fermionic current and does not transform as a gauge field. This Lagrangian does not have any gauge symmetry. Besides the lacks the kinetic term for the Yang-Mills field, the theory given by above Lagrangian is identical with the massive Yang-Mills theory.

The gauge invariant version of the Lagrangian is obtained, by introducing the Stueckelberg field θ which identified with the BF field as shown in [44], as

$$\mathcal{L}_{Th''} = \bar{\psi}^a i \gamma^\mu (\partial_\mu - \frac{i}{\sqrt{N}} A_\mu) \psi^a - m \bar{\psi}^a \psi^a + \frac{1}{2G} (A_\mu - \sqrt{N} \partial_\mu \theta)^2. \quad (3)$$

The original Thirring model can be assumed as the gauge fixed version of this gauge invariant Lagrangian [45], which possesses the following $U(1)$ gauge symmetry:

$$\delta \psi_a = (e^{i\phi} - 1) \psi_a, \quad (4)$$

$$\delta A_\mu = \sqrt{N} \partial_\mu \phi, \quad (5)$$

$$\delta \theta = \phi. \quad (6)$$

Here, ϕ denotes a fictitious Nambu-Goldstone boson field, which has to be absorbed into the longitudinal component of A_μ .

To quantize covariantly a gauge invariant theory, we need to break the local gauge invariance. It removes the fictitious degrees of freedom associated with the theory. This can be achieved by restricting the gauge fields by a general gauge condition, $\Omega = \mathcal{F}[A, \theta] = 0$. This can be incorporated at the level of Lagrangian by adding following linearized gauge-fixing and ghost terms to the classical action,

$$\mathcal{L}_{GF+FP} = B \mathcal{F}[A, \theta] + \frac{\xi}{2} B^2 + i \bar{C} \left(\frac{\delta \mathcal{F}[A, \theta]}{\delta A_\mu} \partial_\mu + \frac{1}{\sqrt{N}} \frac{\delta \mathcal{F}[A, \theta]}{\delta \theta} \right) C, \quad (7)$$

where B is a Nakanishi-Lautrup type multiplier field and ξ is an arbitrary gauge parameter.

A. Lorentz gauge

Now for a particular choice, so-called Lorentz gauge, $\mathcal{F}[A, \theta] = \partial^\mu A_\mu$, the above gauge fixed Lagrangian is alleviated to,

$$\mathcal{L}_{GF+FP}^L = B \partial^\mu A_\mu + \frac{\xi}{2} B^2 + i \bar{C} [\partial_\mu \partial^\mu] C. \quad (8)$$

The effective action for Thirring model in Lorentz gauge is given by

$$\begin{aligned} \mathcal{L}_{Th''} + \mathcal{L}_{GF+GH}^L = & \bar{\psi}^a i\gamma^\mu (\partial_\mu - \frac{i}{\sqrt{N}} A_\mu) \psi^a - m \bar{\psi}^a \psi^a + \frac{1}{2G} (A_\mu - \sqrt{N} \partial_\mu \theta)^2 + B \partial^\mu A_\mu \\ & + \frac{\xi}{2} B^2 + i\bar{C} \partial_\mu \partial^\mu C. \end{aligned} \quad (9)$$

Here, we observe that the Stueckelberg field (BF) θ is coupled to field except for $\xi = 0$ (Landau gauge).

This action is invariant under following BRST transformation:

$$\begin{aligned} \delta_b A_\mu(x) &= -\partial_\mu C \eta, \\ \delta_b B &= 0, \quad \delta_b C = 0, \\ \delta_b \bar{C} &= iB\eta, \quad \delta_b \theta = -\frac{1}{\sqrt{N}} C \eta, \\ \delta_b \psi^j(x) &= \frac{i}{\sqrt{N}} C \psi^j \eta, \end{aligned} \quad (10)$$

where η is an infinitesimal Grassmann parameter.

B. R_ξ gauge

For another important choice of gauge, $\mathcal{F}[A, \theta] = \partial_\mu A^\mu + \sqrt{N} \frac{\xi}{G} \theta$, so-called R_ξ gauge, the gauge fixed Lagrangian is given by

$$\mathcal{L}_{GF+FP}^R = B(\partial_\mu A^\mu + \sqrt{N} \frac{\xi}{G} \theta) + \frac{\xi}{2} B^2 + i\bar{C} \left[\partial_\mu \partial^\mu + \frac{\xi}{G} \right] C. \quad (11)$$

Thus, the Faddeev-Popov effective action for Thirring model in R_ξ gauge is given by

$$\begin{aligned} \mathcal{L}_{Th''} + \mathcal{L}_{GF+GH}^R = & \bar{\psi}^a i\gamma^\mu (\partial_\mu - \frac{i}{\sqrt{N}} A_\mu) \psi^a - m \bar{\psi}^a \psi^a + \frac{1}{2G} (A_\mu - \sqrt{N} \partial_\mu \theta)^2 + B(\partial_\mu A^\mu + \sqrt{N} \frac{\xi}{G} \theta) \\ & + \frac{\xi}{2} B^2 + i\bar{C} \left[\partial_\mu \partial^\mu + \frac{\xi}{G} \right] C. \end{aligned} \quad (12)$$

This, further, reduces to,

$$\begin{aligned} \mathcal{L}_{Th''} + \mathcal{L}_{GF+GH}^R = & \bar{\psi}^a i\gamma^\mu (\partial_\mu - \frac{i}{\sqrt{N}} A_\mu) \psi^a - m \bar{\psi}^a \psi^a - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 + \frac{1}{2G} A_\mu^2 \\ & - \frac{1}{2} \frac{N\xi}{G^2} \theta^2 + \frac{N}{2G} (\partial_\mu \theta)^2 + i\bar{C} \left[\partial_\mu \partial^\mu + \frac{\xi}{G} \right] C. \end{aligned} \quad (13)$$

Here, we see that the Stueckelberg field θ is completely decoupled independently of ξ . The effective action, in R_ξ gauge, is also invariant under the same set of BRST transformation (10).

III. EXTENDED BRST TRANSFORMATION

In this section, we derive the extended BRST formulation, by making the parameter finite and field dependent, known as FFBRST transformation, at a general ground. We, first, define BRST transformation of a generic field $\phi(x)$ as follows:

$$\phi(x) \longrightarrow \phi'(x) = \phi(x) + s_b \phi(x) \eta, \quad (14)$$

where $s_b\phi$ refers to Slavnov variation and η is an infinitesimal anti-commuting global parameter. It is well known that, under such transformation, the path integral measure as well as effective action remain invariant [15].

Now, we interpolate a continuous parameter ($\kappa; 0 \leq \kappa \leq 1$) through the fields $\phi(x)$ such that the $\phi(x, \kappa = 0) = \phi(x)$ is the original field and, however, $\phi(x, \kappa = 1) = \phi'(x) = \phi(x) + s_b\phi(x)\Theta[\phi]$ is the FFBRST transformed field, where $\Theta[\phi]$ is finite field-dependent parameter. Such FFBRST transformation is justified by the following infinitesimal field-dependent BRST transformation: [19]

$$\frac{d\phi(x, \kappa)}{d\kappa} = s_b\phi(x, \kappa)\Theta'[\phi(x, \kappa)]. \quad (15)$$

Now, integrating the above equation w. r. to κ from 0 to 1, we get the FFBRST transformation,

$$\delta_b\phi(x) = \phi'(x) - \phi(x) = s_b\phi(x)\Theta[\phi(x)], \quad (16)$$

where the finite field-dependent parameter is given by

$$\Theta[\phi] = \Theta'[\phi] \frac{\exp f[\phi] - 1}{f[\phi]}, \quad (17)$$

and $f[\phi]$ is given by

$$f[\phi] = \sum_i \int d^4x \frac{\delta\Theta'[\phi]}{\delta\phi_i(x)} s_b\phi_i(x). \quad (18)$$

The FFBRST transformations, with field-dependent parameter, are also symmetry of the effective action but the cost we pay is that, they are no more nilpotent. Contrary to usual BRST symmetry, they do not leave the functional measure invariant. Eventually, the path integral measure under such transformation changes non-trivially, leading to a local Jacobian in the functional integration. So our goal here is to compute the explicit Jacobian of the functional measure, under such a transformations.

A. Jacobian for field-dependent BRST transformation

To compute the Jacobian for path integral measure, under the FFBRST transformation with an arbitrary parameter Θ , we first define the generating functional for the Thirring model described by an effective action $S_{Th}^{F\bar{P}}[\phi]$ as follows,

$$Z[0] = \int \mathcal{D}\phi e^{iS_{Th}^{F\bar{P}}[\phi]}, \quad (19)$$

where $\mathcal{D}\phi$ refers the full functional measure. Furthermore, we write the functional measure under the action of FFBRST transformation as follows [19]

$$\mathcal{D}\phi = J(\kappa)\mathcal{D}\phi(\kappa) = J(\kappa + d\kappa)\mathcal{D}\phi(\kappa + d\kappa). \quad (20)$$

Since, this transformation is infinitesimal, so the transformation from $\phi(\kappa)$ to $\phi(\kappa + d\kappa)$ can, further, be written as [19]

$$\frac{J(\kappa)}{J(\kappa + d\kappa)} = \sum_{\phi} \pm \frac{\delta\phi(\kappa + d\kappa)}{\delta\phi(\kappa)}, \quad (21)$$

where the $+$ sign is used for bosonic fields and $-$ is used for the fermionic fields. Utilizing Taylor expansion around κ in the above expression, we get the following identification [19]:

$$\frac{1}{J} \frac{dJ}{d\kappa} d\kappa = -d\kappa \int d^Dx \sum_{\phi} \pm s_b\phi(x, \kappa) \frac{\delta\Theta'[\phi(x, \kappa)]}{\delta\phi(x, \kappa)}. \quad (22)$$

This reduces to

$$\frac{d \ln J[\phi]}{d\kappa} = - \int d^D x \sum_{\phi} \pm s_b \phi(x, \kappa) \frac{\delta \Theta'[\phi(x, \kappa)]}{\delta \phi(x, \kappa)}. \quad (23)$$

To attain the expression for the finite Jacobian, from the (above) infinitesimal one, we integrate it over κ with limits from 0 to 1. This leads to a logarithmic series,

$$\begin{aligned} \ln J[\phi] &= - \int_0^1 d\kappa \int d^D x \sum_{\phi} \pm s_b \phi(x, \kappa) \frac{\delta \Theta'[\phi(x, \kappa)]}{\delta \phi(x, \kappa)}, \\ &= - \left(\int d^D x \sum_{\phi} \pm s_b \phi(x) \frac{\delta \Theta'[\phi(x)]}{\delta \phi(x)} \right). \end{aligned} \quad (24)$$

Now, exponentiating the above relation leads to the expression for Jacobian for functional measure, under FFBRST transformation with an arbitrary field dependent parameter Θ' , as follows

$$J[\phi] = \exp \left(- \int d^D x \sum_{\phi} \pm s_b \phi(x) \frac{\delta \Theta'[\phi(x)]}{\delta \phi(x)} \right). \quad (25)$$

Here, we see that the Jacobian (25) extends the Faddeev-Popov action (within functional integral) of the theory, given in (19), as following:

$$\int \mathcal{D}\phi' e^{iS_{Th}^{FP}[\phi']} = \int J[\phi] \mathcal{D}\phi e^{iS_{Th}^{FP}[\phi]} = \int \mathcal{D}\phi e^{i(S_{Th}^{FP}[\phi] + i \int d^D x (\sum_{\phi} \pm s_b \phi \frac{\delta \Theta'}{\delta \phi}))}. \quad (26)$$

We will notice that the Jacobian amounts precise change in the BRST exact part of the action, so, the dynamics of the theory does not change as the BRST exact part of a BRST invariant function does not alter the dynamics of the theory at cohomological level.

IV. CONNECTION OF LORENTZ GAUGE TO R_{ξ} GAUGE

In this section, we illustrate the results of section III with an specific example. Following the methodology discussed above, we first construct the FFBRST transformation for Thirring model,

$$\begin{aligned} \delta_b A_{\mu}(x) &= -\partial_{\mu} C \Theta[\phi], \\ \delta_b B &= 0, \quad \delta_b C = 0, \\ \delta_b \bar{C} &= i B \Theta[\phi], \\ \delta_b \theta &= -\frac{1}{\sqrt{N}} C \Theta[\phi], \\ \delta_b \psi^j(x) &= \frac{i}{\sqrt{N}} C \psi^j \Theta[\phi], \end{aligned} \quad (27)$$

where $\Theta[\phi]$ is an arbitrary finite field-dependent parameter satisfying $\Theta^2 = 0$. Now, we construct an specific Θ , described in terms of Θ' , to see the effect of FFBRST transformation in Thirring model. This is given by

$$\Theta'[\phi] = \int d^D x \left[\bar{C} \sqrt{N} \frac{\xi}{G} \theta \right]. \quad (28)$$

For this choice of parameter, we calculate the Jacobian for functional measure

$$J[\phi] = \exp \left(i \int d^D x \left[B \sqrt{N} \frac{\xi}{G} \theta + i \bar{C} \frac{\xi}{G} C \right] \right), \quad (29)$$

where (25) is utilized.

Here, we observe that the Jacobian contributes to the unphysical (BRST exact) part of the action. This Jacobian modifies the expression of generating functional in Lorentz gauge as follows

$$\int \mathcal{D}\phi' e^{i \int d^D x (\mathcal{L}_{Th''} + \mathcal{L}_{GF+GH}^L)[\phi']} \xrightarrow{FFBRST} \int \mathcal{D}\phi e^{i \int d^D x (\mathcal{L}_{Th''} + \mathcal{L}_{GF+GH}^R)[\phi]}, \quad (30)$$

where the final expression is nothing but the generating functional in R_ξ gauge. Such modification does not alter the theory because the extra pieces, due to Jacobian, attribute to the BRST exact part of the action. Though we have shown the connection of two specific gauges, this results are valid for any arbitrary pair of gauges. Suppose, we choose a parameter $\Theta'[A, \theta, \bar{C}] = \int d^D x [\bar{C} (\mathcal{F}_1[A, \theta] - \mathcal{F}_2[A, \theta])]$, where \mathcal{F}_1 and \mathcal{F}_2 are two arbitrary gauges, then, the Jacobian will map the generating functional corresponding to these two gauges. Thus, we see that the two well studied gauges of Thirring model are related to each other. This is shown with the helps of extended BRST transformation, with a particular parameter of transformation.

V. BV FORMULATION AND FFBRST SYMMETRY

In the BV formulation, the generating functional of Thirring model (in Lorentz gauge), by introducing antifields ϕ^* corresponding to the all fields $\phi(\equiv A_\mu, \bar{C}, C, B, \theta)$ with opposite statistics, is given by

$$Z_L = \int \mathcal{D}\phi e^{i \int d^4 x (\mathcal{L}_{Th''} + \mathcal{L}_{GF+GH}^L[\phi, \phi^*])}. \quad (31)$$

This can, further, be written in compact form as

$$Z_L = \int \mathcal{D}\phi e^{i W_{\Psi_L}[\phi, \phi^*]}, \quad (32)$$

where $W_{\Psi_L}[\phi, \phi^*]$ is an extended quantum action in Lorentz gauge. The generating functional does not depend on the choice of gauge-fixing fermion [36]. The extended quantum action for Thirring model, $W_{\Psi_L}[\phi, \phi^*]$, satisfies the following mathematically rich relation, called the quantum master equation [15],

$$\Delta e^{i W_{\Psi_L}[\phi, \phi^*]} = 0 \quad \text{with} \quad \Delta \equiv \frac{\partial_r}{\partial \phi} \frac{\partial_r}{\partial \phi^*} (-1)^{\epsilon+1}. \quad (33)$$

The antifields, which get identified with gauge-fixing fermion in Lorentz gauge $\Psi_L = -i\bar{C} \left(\partial_\mu A^\mu + \frac{\xi}{2} B \right)$, are

$$\begin{aligned} A_\mu^* &= \frac{\delta \Psi_L}{\delta A^\mu} = i \partial_\mu \bar{C}, \\ \bar{C}^* &= \frac{\delta \Psi_L}{\delta \bar{C}} = -i \left(\partial_\mu A^\mu + \frac{\xi}{2} B \right), \\ C^* &= \frac{\delta \Psi_L}{\delta C} = 0, \quad \theta^* = \frac{\delta \Psi_L}{\delta \theta} = 0. \end{aligned} \quad (34)$$

Similarly, the generating functional for Thirring model in R_ξ gauge is defined, compactly, as

$$\begin{aligned} Z_R &= \int \mathcal{D}\phi e^{i \int d^4 x (\mathcal{L}_{Th''} + \mathcal{L}_{GF+GH}^R[\phi, \phi'^*])}, \\ &= \int \mathcal{D}\phi e^{i W_{\Psi_R}[\phi, \phi'^*]}. \end{aligned} \quad (35)$$

The following expression for antifields, in the case of R_ξ gauge, are obtained

$$\begin{aligned} A'^\star_\mu &= \frac{\delta\Psi_R}{\delta A^\mu} = i\partial_\mu\bar{C}, \\ \bar{C}'^\star &= \frac{\delta\Psi_R}{\delta\bar{C}} = -i\left(\partial_\mu A^\mu + \sqrt{N}\frac{\xi}{G}\theta + \frac{\xi}{2}B\right), \\ C'^\star &= \frac{\delta\Psi_R}{\delta C} = 0, \quad \theta'^\star = \frac{\delta\Psi_R}{\delta\theta} = -i\sqrt{N}\frac{\xi}{G}\bar{C}, \end{aligned} \quad (36)$$

where $\Psi_R = -i\bar{C}\left(\partial_\mu A^\mu + \sqrt{N}\frac{\xi}{G}\theta + \frac{\xi}{2}B\right)$ is utilized. To connect Lorentz and R_ξ gauges in BV formulation, we construct the following infinitesimal field-dependent parameter $\Theta'[\phi]$

$$\Theta'[\phi] = i \int d^D y [\bar{C}\bar{C}'^\star - \bar{C}C'^\star]. \quad (37)$$

The Jacobian of the path integral measure in the generating functional, for this parameter, is computed utilizing relation (25). The resulting Jacobian factor changes the quantum action as

$$W_{\Psi_L}[\phi, \phi^\star] \xrightarrow{FFBRST} W_{\Psi_R}[\phi, \phi^\star]. \quad (38)$$

This reflects the validity of result at quantum levels also. Hence, we conclude that the FFBRST transformations connect two different solutions of quantum master equation of the Thirring model.

VI. CONCLUSION

From the effective potential points of view, the existence of the second order phase transition associated with the spontaneous breakdown of the chiral symmetry in the D ($2 < D < 4$) dimensional Thirring model has been analysed. And the explicit critical number of flavors has derived as a function of the four-fermion coupling constant. The Thirring model as a gauge theory, by introducing the Stueckelberg field as a BF field, is well studied. In this context, without gauge interactions the Thirring model is identified with the gauge-fixed version of a gauge theory and has the well known BRST symmetry even after the gauge-fixing.

In this paper, we have considered the gauge invariant as well as renormalizable Thirring model in D ($2 < D < 4$) dimensions. Since, the gauge invariance possesses the unphysical degrees of freedom. According to standard quantization procedure, we have to remove them by breaking the local gauge invariance. This can be achieved by fixing an appropriate gauge. We have discussed, the well-studied, Lorentz and R_ξ gauges in this context. The remarkable properties of these gauges in Thirring model are that, in R_ξ gauge, the Stueckelberg (BF) field θ is completely decoupled to the massive vector boson independently of ξ and makes the computations simple. However, in Lorentz gauge, the Stueckelberg field θ is decoupled to the massive vector boson only if $\xi = 0$. In this sense, R_ξ gauge is more acceptable for the model. To map these gauges, we have extended the BRST symmetry by making the transformation parameter finite and field dependent. Under such transformation, the path integral measure is not unchanged, rather it changes in a non-trivial way. The Jacobian of path integral measure depends, explicitly, on field-dependent parameter. We compute the Jacobian for an arbitrary parameter, to be valid at a general ground. However, for a specific choice of parameter, we have illustrated that the Jacobian connects the Lorentz gauge to R_ξ gauge. Though we have established a connection for a particular set of gauges, this formalism would be valid for connecting any two set of gauges. We have computed the extended quantum action as well as quantum master equation, utilizing BV formulation. Further, we have shown a mapping between the two different solutions of quantum master equation with the help of FFBRST transformation. Since, the analysis of the Thirring model, as the gauged non-linear sigma model, is given from the viewpoint of the constrained system, which implies that the present

investigation might be useful from the perspectives of non-linear sigma model.

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